Olympiad News

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In the last column, it was mentioned that Australia's International Mathematics Olympiad (IMO) team would be announced in early June. At the time we had high hopes that Alexander Chua (Year 11, Christ Church Grammar School) might make the team, but the competition turned out to be particularly intense this year, and he missed out completely, after having made reserve last year. Alexander is very determined to be in the team next year when it will be in Colombia. The IMO was held 4th -16th July in Mar Del Plata, Argentina, this year. At the time of writing the results are just in. All team members medalled with 4 Bronze and 2 Silver, but nevertheless the team dropped two places to 27th (last year's mix of medals was 3 of each of Bronze and Silver). For full results, see:

http://www.imo-official.org/results.aspx

and, by the time of printing of this article, the Australian summary will be at:

http://www.amt.edu.au/imo2012.html

Last year I mentioned that at the IMO site, there is a Hall of Fame, and noted that Number 1, was Lisa Sauermann of Germany, who having attended 5 IMOs, had a haul of 4 Gold and 1 Silver. Well that didn't last long. She's Number 2, now, behind Teodor von Burg, who having attended 6 IMOs, pips Lisa by a Bronze. Nevertheless, Lisa's Gold medal achievements were significantly higher than those achieved by Teodor.

Tournament of the Towns (TT), Northern Spring round, for 2011-12, was held on Saturday, 5 May (O Level paper) and Saturday, 11 May (A level paper). Recall that the Tournament of the Towns is an "invitation-only" mathematics competition, with a 4-hour O paper of five questions, and a 5-hour A paper with seven questions. A student's score on a paper is the highest total for their attempts at three of the questions. A student's overall score for the TT round is the better result of the two papers. The three highest ranked juniors and the two highest ranked seniors have had their papers forwarded to Moscow for a more rigorous marking; and hopefully they will receive a Diploma from the Russian Academy of Sciences, to go with their certificate from the Australian Mathematics Trust. A summary of these results in order of rank is below.

Junior Student	*Year*	*School*	*Result*	*WA Rank*
Devin He	8	Christ Church GS	Distinction	1
Henry Yoo	10	Perth Modern School	Distinction	2
Satthya Krishnasivam	10	Perth Modern School	Distinction	3
Ananthu Koloth	8	Christ Church GS	Credit	4
Nicholas Pizzino	8	Christ Church GS	Participation	5
Albert Qiu	9	Christ Church GS	Participation	6
Nicholas Lim	8	Christ Church GS	Participation	7
Vandit Trivedi	8	Christ Church GS	Participation	8

Senior Student	*Year*	*School*	*Result*	*WA Rank*
Alexander Chua	11	Christ Church GS	High Distinction	1
Edward Yoo	11	All Saints' College	Distinction	2
Conway Li	11	Perth Modern School	Credit	3
Aaron Hurst	12	Home School	Credit	4
Jack Cooper	11	Hale School	Participation	5
Diffy Zhou	11	Perth Modern School	Participation	6
Ciaran Murray	11	Trinity College	Participation	=7
Joseph Thompson	11	Perth Modern School	Participation	=7

The next Olympiad level events are the Senior Mathematics Contest (SMC) and Australian Intermediate Mathematical Olympiad (AIMO) to be held on 14 and 16 August, respectively.

Finally, let us close with the following lovely problem from the recent Senior O Level TT.

Question 3 (TT, Northern Spring 2012, Senior O Level):
Consider the points of intersections of the graphs $y = \cos x$ and $x = 100\cos(100y)$ for which both coordinates are positive. Let a be the sum of their x-coordinates and b be the sum of their y-coordinates. Determine the value of a/b.

Solution. Let $X = x/10$ and let $Y = 10y$. Then the graphs become the symmetric pair

$$Y = 10\cos(10X) \text{ and } X = 10\cos(10Y).$$

Now X and Y are both positive if and only if both x and y are positive. The transformation from (x, y) coordinates to (X, Y) coordinates is a contraction in the x direction and a dilation in the y direction, that preserves the points of intersection, i.e. if (x_i, y_i) is an intersection of the given graphs, then $(X_i, Y_i) = (x_i/10, 10y_i)$ is the corresponding intersection point of the graphs in (X, Y) coordinates. Let A be the sum of the X-coordinates and let B be the sum of the Y-coordinates of the points of intersection of the graphs in (X, Y) coordinates, for which both X and Y are positive. Then

$$A = \sum_i X_i = \sum_i (x_i/10) = \frac{1}{10}\sum_i x_i = \frac{a}{10}$$
$$B = \sum_i Y_i = \sum_i (10y_i) = 10\sum_i y_i = 10b$$

Hence $a = 10A$ and $b = B/10$ and, since the equations in X and Y are symmetric, $A = B$. So,

$$\frac{a}{b} = \frac{10A}{B/10} = 100.$$

Remark. This question is reminiscent of a problem from last year at this time. The key thing there was exploitation of symmetry. Here we didn't have it, initially, but noticing that we can get symmetry is what cracks the problem.